Physics Letters B 750 (2015) 117–121

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Chern–Simons supergravity in $D = 3$ and Maxwell superalgebra

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ARTICLE INFO

Article history:
Received 15 July 2015
Received in revised form 19 August 2015
Accepted 2 September 2015
Available online 5 September 2015
Editor: M. Cvetič

ABSTRACT

We present the construction of the $D = 3$ Chern–Simons supergravity action without cosmological constant from the minimal Maxwell superalgebra $s\mathcal{M}_3$. This superalgebra contains two Majorana fermionic charges and can be obtained from the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ superalgebra using the abelian semigroup expansion procedure. The components of the Maxwell invariant tensor are explicitly derived.

1. Introduction

There is a particular interest in (super)gravity theories in modifying the Poincaré symmetries to bigger ones. A well known enlargement of the Poincaré algebra is the Maxwell algebra $\mathcal{M}$ which is obtained by adding a constant electromagnetic field background to the Minkowski spacetime [1,2]. This algebra is characterized by the introduction of tensorial generators Z_{ab} which modify the commutation relation of the translation generators P_a,

$$[P_a, P_b] = Z_{ab}. \tag{1}$$

Based on the $D = 4$ Maxwell symmetries, it was recently shown an alternative way to introduce a generalized cosmological term to a gravity action [3]. Furthermore, it was recently pointed out that the Maxwell type algebras allow to recover General Relativity from Chern–Simons (CS) and Born–Infeld (BI) gravity theories [4–7].

Interestingly, the supersymmetric extension of the Maxwell algebra describes the geometry of a four dimensional $\mathcal{N} = 1$ superspace in the presence of a constant abelian supersymmetric gauge field background [8]. This modifies the superMinkowski space into the superMaxwell space. In particular, the $D = 4$ minimal Maxwell superalgebra introduced in [8] contains the usual Maxwell algebra as subalgebra. This minimal Maxwell superalgebra and its generalizations have been extensively studied using the expansion methods in Refs. [9,10]. These superalgebras have the particularity to have more than one spinor charge and can be viewed as the generalization of the D'Auria–Fré superalgebra and the Green algebras introduced respectively in Refs. [11,12].

In particular, the minimal Maxwell superalgebra is obtained after a resonant S-expansion of $\mathfrak{osp}(4|1)$ superalgebra [10]. Subsequently, as shown in Ref. [13], pure supergravity can be derived as a MacDowell–Mansouri like action from the Maxwell symmetries.

The S-expansion method is a powerful tool in order to derive new Lie (super)algebras and build new (super)gravity theories. Basically, it consists in combining the structure constants of a Lie (super)algebra $\mathfrak{g}$ with the multiplication law of a semigroup S [14]. A very useful advantage of this procedure is that it provides with an invariant tensor for the S-expanded (super)algebra $\mathfrak{G} = S \times \mathfrak{g}$ in terms of an invariant tensor for the original (super)algebra $\mathfrak{g}$. In particular, the invariant tensor is a crucial ingredient in the construction of (super)gravity actions. Some interesting applications of the S-expansion method in (super)gravity theories can be found in Refs. [4–7,13,15–17].

An interesting formalism which allows to construct a gauge theory of supergravity in odd dimensions is the Chern–Simons approach. A CS gravity theory has the advantage to be a "gauge" theory of gravity whose spin connection and vielbein can be seen as independent fields. In particular, a good candidate to describe a three-dimensional CS supergravity theory with a cosmological constant is the *AdS* supergroup. The most generalized supersymmetric extension of the three-dimensional *AdS* algebra is given by the direct product [18]

$$\mathfrak{osp}(2|p) \otimes \mathfrak{osp}(2|q)\,, \tag{2}$$

describing a (p, q)-type *AdS*–Chern–Simons supergravity in presence of a cosmological constant. The CS supergravity action is con-

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http://dx.doi.org/10.1016/j.physletb.2015.09.005

structed out of the connection one-form A associated to the *AdS* supergroup as follows [19,20]

$$S=k\int\left\langle A\left(dA+\frac{2}{3}A^{2}\right)\right\rangle, \tag{3}$$

where $\langle\cdots\rangle$ denotes the invariant tensor.

Interestingly, the $\mathfrak{osp}(2|p)\otimes\mathfrak{osp}(2|q)$ superalgebra allows to construct a non-minimal three-dimensional *AdS* CS supergravity theory. In particular, the minimal *AdS* CS supergravity is obtained when $p=1$ and $q=0$ ($\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$) [21]. As was pointed out in Ref. [18], the presence of $\mathcal{N}=p+q$ supersymmetries allows to introduce CS terms related to the $O(p)\otimes O(q)$ gauge symmetry.

The purpose of this paper is to construct a $D=3$ minimal Maxwell–Chern–Simons supergravity action. To this aim, the S-expansion of the $\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$ superalgebra is considered in order to derive the minimal Maxwell superalgebra $s\mathcal{M}_3$. The procedure considered here allows to find the non-vanishing components of a Maxwell invariant tensor which are required to construct a CS action. The CS formalism used here represents a toy model in order to approach problems present in higher dimensions or in higher $\mathcal{N}$-extended supergravity theories.

This work is organized as follows: in Section 2 we briefly review the CS supergravity theory for the $\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$ superalgebra. Section 3 contains our main results. We obtain the minimal Maxwell superalgebra $s\mathcal{M}_3$ from the $\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$ superalgebra using the S-expansion procedure. The components of a superMaxwell invariant tensor are presented and a CS supergravity action is constructed. Section 4 concludes the work with some comments about the Maxwell supersymmetries and possible developments.

2. $D=3$ Chern–Simons supergravity theory and *AdS* superalgebra

In this section we briefly review the construction of the Chern–Simons supergravity action in $D=3$ for the *AdS* superalgebra, $\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$. The (anti)commutation relations for this superalgebra are given by

$$\left[\tilde{J}_{ab},\tilde{J}_{cd}\right]=\eta_{bc}\tilde{J}_{ad}-\eta_{ac}\tilde{J}_{bd}-\eta_{bd}\tilde{J}_{ac}+\eta_{ad}\tilde{J}_{bc}, \tag{4}$$

$$\left[\tilde{J}_{ab},\tilde{P}_{c}\right]=\eta_{bc}\tilde{P}_{a}-\eta_{ac}\tilde{P}_{b}, \tag{5}$$

$$\left[\tilde{P}_{a},\tilde{P}_{b}\right]=\tilde{J}_{ab}, \tag{6}$$

$$\left[\tilde{P}_{a},\tilde{Q}_{\alpha}\right]=-\frac{1}{2}\left(\Gamma_{a}\tilde{Q}\right)_{\alpha}, \tag{7}$$

$$\left[\tilde{J}_{ab},\tilde{Q}_{\alpha}\right]=-\frac{1}{2}\left(\Gamma_{ab}\tilde{Q}\right)_{\alpha}, \tag{8}$$

$$\left\{\tilde{Q}_{\alpha},\tilde{Q}_{\beta}\right\}=-\frac{1}{2}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}\tilde{J}_{ab}-2\left(\Gamma^{a}C\right)_{\alpha\beta}\tilde{P}_{a}\right], \tag{9}$$

where $\tilde{J}_{ab}$, $\tilde{P}_{a}$ and $\tilde{Q}_{\alpha}$ are the generators of Lorentz transformations, the *AdS* boost and supersymmetry, respectively. Here C stands for the charge conjugation matrix, Γ_a are Dirac matrices and $\Gamma_{ab}=\frac{1}{2}\left[\Gamma_a,\Gamma_b\right]$.

The Chern–Simons action in $(2+1)$ dimensions [19,20] is given by

$$S_{CS}^{(2+1)}=k\int\left\langle A\left(dA+\frac{2}{3}A^{2}\right)\right\rangle. \tag{10}$$

Here, A corresponds the one-form gauge connection for the $\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$ superalgebra

$$A=\frac{1}{2}\omega^{ab}\tilde{J}_{ab}+\frac{1}{l}e^{a}\tilde{P}_{a}+\frac{1}{\sqrt{l}}\psi^{\alpha}\tilde{Q}_{\alpha}, \tag{11}$$

whose associated curvature two-form $F=dA+A\wedge A$ is

$$F=F^{A}T_{A}=\frac{1}{2}\mathcal{R}^{ab}\tilde{J}_{ab}+\frac{1}{l}R^{a}\tilde{P}_{a}+\frac{1}{\sqrt{l}}\Psi^{\alpha}\tilde{Q}_{\alpha}, \tag{12}$$

where

$$\mathcal{R}^{ab}=d\omega^{ab}+\omega^{a}{}_{c}\omega^{cb}+\frac{1}{l^{2}}e^{a}e^{b}+\frac{1}{2l}\bar{\psi}\Gamma^{ab}\psi,$$
$$R^{a}=de^{a}+\omega^{a}{}_{b}e^{b}-\frac{1}{2}\bar{\psi}\Gamma^{a}\psi,$$
$$\Psi=\nabla\psi=d\psi+\frac{1}{4}\omega_{ab}\Gamma^{ab}\psi+\frac{1}{2l}e^{a}\Gamma_{a}\psi.$$

In (10) the bracket $\langle\cdots\rangle$ stands for the non-vanishing components of an invariant tensor for the $\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$ superalgebra in $(2+1)$-dimensions:

$$\left\langle\tilde{J}_{ab}\tilde{J}_{cd}\right\rangle=\mu_{0}\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{13}$$

$$\left\langle\tilde{J}_{ab}\tilde{P}_{c}\right\rangle=\mu_{1}\epsilon_{abc}, \tag{14}$$

$$\left\langle\tilde{P}_{a}\tilde{P}_{b}\right\rangle=\mu_{0}\eta_{ab}, \tag{15}$$

$$\left\langle\tilde{Q}_{\alpha}\tilde{Q}_{\beta}\right\rangle=\left(\mu_{0}-\mu_{1}\right)C_{\alpha\beta}, \tag{16}$$

where μ_0 and μ_1 are arbitrary constants.

Considering (13)–(16) and the one-form connection (11), the CS action (10) for the $\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$ superalgebra can be written as

$$\begin{aligned}S_{CS}^{(2+1)}=k\int_{M}&\frac{\mu_{0}}{2}\left(\omega^{a}{}_{b}d\omega^{b}{}_{a}+\frac{2}{3}\omega^{a}{}_{c}\omega^{c}{}_{b}\omega^{b}{}_{a}+\frac{2}{l^{2}}e^{a}T_{a}+\frac{2}{l}\bar{\psi}\Psi\right)\\&+\frac{\mu_{1}}{l}\left(\epsilon_{abc}\left(R^{ab}e^{c}+\frac{1}{3l^{2}}e^{a}e^{b}e^{c}\right)-\bar{\psi}\Psi\right)\\&-d\left(\frac{\mu_{1}}{2l}\epsilon_{abc}\omega^{ab}e^{c}\right)\end{aligned} \tag{17}$$

where $T^{a}=de^{a}+\omega^{a}{}_{b}e^{b}$ is the torsion 2-form. This action describes the most general $\mathcal{N}=1$, $D=3$ Chern–Simons supergravity action (with cosmological constant) for the *AdS* supergroup [21].

3. The minimal Maxwell superalgebra and CS supergravity action

The abelian semigroup expansion method (S-expansion) is a powerful tool in order to obtain new Lie (super)algebras from known ones [14,22]. The S-expansion procedure consists in combining the multiplication law of a semigroup S with the structure constants of a Lie algebra $\mathfrak{g}$. The new Lie algebra $\mathfrak{G}=S\times\mathfrak{g}$ is called the S-expanded algebra.

In this section, we show that a three-dimensional minimal Maxwell superalgebra $s\mathcal{M}_3$ can be derived from the $\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$ superalgebra using the S-expansion procedure with a particular choice of a semigroup S. The result obtained here will be useful in the construction of a Maxwell–Chern–Simons supergravity in $D=3$ which we shall approach in the next subsection.

A necessary step before applying the S-expansion method consists in considering a decomposition of the original algebra $\mathfrak{g}=\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)$ in subspaces V_p,

$$\mathfrak{g}=\mathfrak{osp}(2|1)\otimes\mathfrak{sp}(2)=V_{0}\oplus V_{1}\oplus V_{2} \tag{18}$$

where V_0 corresponds to the Lorentz subalgebra which is generated by $\tilde{J}_{ab}$, V_1 corresponds to the supersymmetry translation generated by $\tilde{Q}_{\alpha}$ and V_2 corresponds to the AdS boost generated by $\tilde{P}_{a}$. The subspace structure may be written as

$$[V_0, V_0] \subset V_0, \quad [V_1, V_1] \subset V_0 \oplus V_2,$$
$$[V_0, V_1] \subset V_1, \quad [V_1, V_2] \subset V_1,$$
$$[V_0, V_2] \subset V_2, \quad [V_2, V_2] \subset V_0. \tag{19}$$

The next step consists in finding a subset decomposition of a semigroup S which is "resonant" with respect to the subspace structure (19). Let us consider $S_E^{(4)} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$ as the relevant finite abelian semigroup whose elements are dimensionless and obey the multiplication law

$$\lambda_\alpha \lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{when } \alpha+\beta \leq 5, \\ \lambda_5, & \text{when } \alpha+\beta > 5. \end{cases} \tag{20}$$

Here λ_5 plays the role of the zero element of the semigroup $S_E^{(4)}$, so we have for each $\lambda_\alpha \in S_E^{(4)}$, $\lambda_5\lambda_\alpha = \lambda_5 = 0_s$. Let us consider a subset decomposition $S_E^{(4)} = S_0 \cup S_1 \cup S_2$, with

$$S_0 = \{\lambda_0, \lambda_2, \lambda_4, \lambda_5\}, \tag{21}$$
$$S_1 = \{\lambda_1, \lambda_3, \lambda_5\}, \tag{22}$$
$$S_2 = \{\lambda_2, \lambda_4, \lambda_5\}. \tag{23}$$

This subset decomposition is said to be "resonant" since it satisfies the same structure as the subspaces V_p [compare with eqs. (19)]

$$S_0 \cdot S_0 \subset S_0, \quad S_1 \cdot S_1 \subset S_0 \cap S_2,$$
$$S_0 \cdot S_1 \subset S_1, \quad S_1 \cdot S_2 \subset S_1,$$
$$S_0 \cdot S_2 \subset S_2, \quad S_2 \cdot S_2 \subset S_0. \tag{24}$$

Following Theorem IV.2 of Ref. [14], we can say that the superalgebra

$$\mathfrak{G}_R = W_0 \oplus W_1 \oplus W_2, \tag{25}$$

is a resonant subalgebra of $S_E^{(4)} \times \mathfrak{g}$, where

$$W_0 = (S_0 \times V_0) = \left\{\lambda_0 \tilde{J}_{ab}, \lambda_2 \tilde{J}_{ab}, \lambda_4 \tilde{J}_{ab}, \lambda_5 \tilde{J}_{ab}\right\}, \tag{26}$$
$$W_1 = (S_1 \times V_1) = \left\{\lambda_1 \tilde{Q}_\alpha, \lambda_3 \tilde{Q}_\alpha, \lambda_5 \tilde{Q}_\alpha\right\}, \tag{27}$$
$$W_2 = (S_2 \times V_2) = \left\{\lambda_2 \tilde{P}_a, \lambda_4 \tilde{P}_a, \lambda_5 \tilde{P}_a\right\}. \tag{28}$$

Imposing the 0_S-reduction condition,

$$\lambda_5 T_A = 0_s, \tag{29}$$

we find a new Lie superalgebra generated by $\{J_{ab}, P_a, \tilde{Z}_{ab}, Z_{ab}, \tilde{Z}_a, Q_\alpha, \Sigma_\alpha\}$ where these generators can be written as

$$J_{ab} = \lambda_0 \tilde{J}_{ab}, \quad \tilde{Z}_a = \lambda_2 \tilde{P}_a,$$
$$\tilde{Z}_{ab} = \lambda_2 \tilde{J}_{ab}, \quad Q_\alpha = \lambda_1 \tilde{Q}_\alpha,$$
$$Z_{ab} = \lambda_4 \tilde{J}_{ab}, \quad \Sigma_\alpha = \lambda_3 \tilde{Q}_\alpha,$$
$$P_a = \lambda_2 \tilde{P}_a, \tag{30}$$

and satisfy the following (anti)commutation relations

$$[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \tag{31}$$
$$[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \qquad [P_a, P_b] = Z_{ab}, \tag{32}$$
$$[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{33}$$
$$[P_a, Q_\alpha] = -\frac{1}{2} (\Gamma_a \Sigma)_\alpha, \tag{34}$$
$$[J_{ab}, Q_\alpha] = -\frac{1}{2} (\Gamma_{ab} Q)_\alpha, \tag{35}$$
$$[J_{ab}, \Sigma_\alpha] = -\frac{1}{2} (\Gamma_{ab} \Sigma)_\alpha, \tag{36}$$
$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2}\left[\left(\Gamma^{ab} C\right)_{\alpha\beta} \tilde{Z}_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta} P_a\right], \tag{37}$$
$$\{Q_\alpha, \Sigma_\beta\} = -\frac{1}{2}\left[\left(\Gamma^{ab} C\right)_{\alpha\beta} Z_{ab} - 2\left(\Gamma^a C\right)_{\alpha\beta} \tilde{Z}_a\right] \tag{38}$$
$$\left[J_{ab}, \tilde{Z}_{ab}\right] = \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc}, \tag{39}$$
$$\left[\tilde{Z}_{ab}, \tilde{Z}_{cd}\right] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{40}$$
$$\left[J_{ab}, \tilde{Z}_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \qquad \left[\tilde{Z}_{ab}, P_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \tag{41}$$
$$\left[\tilde{Z}_{ab}, Q_\alpha\right] = -\frac{1}{2} (\gamma_{ab} \Sigma)_\alpha, \tag{42}$$
$$\text{others} = 0. \tag{43}$$

Here we have used the multiplication law of the semigroup (20) and the commutation relations of the original superalgebra (4)–(9). The new superalgebra obtained after a 0_S-reduced resonant S-expansion of $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ corresponds to the minimal Maxwell superalgebra $s\mathcal{M}_3$. This superalgebra can be seen as the supersymmetric extension of the generalized Maxwell algebra $g\mathcal{M}$ in $D = 3$ dimensions [10].

Let us note that if we set $\tilde{Z}_{ab} = \tilde{Z}_c = 0$, we obtain the usual minimal Maxwell superalgebra [8]. As was pointed out in Ref. [10], this can be done since the Jacobi identities for the spinorial generators are satisfied due to the gamma matrix identity $(C\gamma^a)_{(\alpha\beta} (C\gamma_a)_{\gamma\delta)} = 0$.

One can see that the minimal Maxwell superalgebra $s\mathcal{M}_3$ contains the Maxwell algebra $\mathcal{M} = \{J_{ab}, P_a, Z_{ab}\}$ and the Lorentz type algebra $\mathcal{L}^{\mathcal{M}} = \{J_{ab}, Z_{ab}\}$ introduced in Ref. [5] as subalgebras.

3.1. Maxwell CS supergravity action in $D = 3$

Here, following the definitions and properties of the S-expansion method, we present the construction of a three-dimensional Maxwell supergravity action in the Chern–Simons formalism.

In order to write down a CS action for the minimal Maxwell superalgebra $s\mathcal{M}_3$ we start from the one-form gauge connection

$$A = \frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{2}\tilde{k}^{ab} \tilde{Z}_{ab} + \frac{1}{2}k^{ab} Z_{ab} + \frac{1}{l}e^a P_a + \frac{1}{l}\tilde{h}^a \tilde{Z}_a + \frac{1}{\sqrt{l}}\psi^\alpha Q_\alpha + \frac{1}{\sqrt{l}}\xi^\alpha \Sigma_\alpha, \tag{44}$$

where the 1-form gauge fields are given in terms of the components of the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ connection $\tilde{e}^a, \tilde{\omega}^{ab}$ and $\tilde{\psi}$:

$$\omega^{ab} = \lambda_0 \tilde{\omega}^{ab}, \quad \tilde{k}^{ab} = \lambda_2 \tilde{\omega}^{ab} \quad k^{ab} = \lambda_4 \tilde{\omega}^{ab}, \quad e^a = \lambda_2 \tilde{e}^a,$$
$$\tilde{h}^a = \lambda_4 \tilde{e}^a, \quad \psi^\alpha = \lambda_1 \tilde{\psi}^\alpha, \quad \xi^\alpha = \lambda_3 \tilde{\psi}^\alpha.$$

The associated curvature two-form $F = dA + A \wedge A$ is given by

$$F = F^A T_A = \frac{1}{2}R^{ab} J_{ab} + \frac{1}{l}R^a P_a + \frac{1}{2}\tilde{F}^{ab} \tilde{Z}_{ab} + \frac{1}{2}F^{ab} Z_{ab} + \frac{1}{l}\tilde{H}^a \tilde{Z}_a + \frac{1}{\sqrt{l}}\Psi^\alpha Q_\alpha + \frac{1}{\sqrt{l}}\Xi^\alpha \Sigma_\alpha, \tag{45}$$

where

$$R^{ab} = d\omega^{ab} + \omega^a{}_c \omega^{cb},$$
$$R^a = de^a + \omega^a{}_b e^b - \frac{1}{2}\bar{\psi}\Gamma^a \psi,$$

$$\tilde{F}^{ab} = d\tilde{k}^{ab} + \omega^{a}{}_{c}\tilde{k}^{cb} - \omega^{b}{}_{c}\tilde{k}^{ca} + \frac{1}{2l}\bar{\psi}\Gamma^{ab}\psi,$$
$$F^{ab} = dk^{ab} + \omega^{a}{}_{c}k^{cb} - \omega^{b}{}_{c}k^{ca} + \tilde{k}^{a}{}_{c}\tilde{k}^{cb} + \frac{1}{l^2}e^{a}e^{b} + \frac{1}{l}\bar{\xi}\Gamma^{ab}\psi,$$
$$\tilde{H}^{a} = d\tilde{h}^{a} + \omega^{a}{}_{b}\tilde{h}^{b} + \tilde{k}^{a}{}_{c}e^{c} - \bar{\xi}\Gamma^{a}\psi,$$
$$\Psi = d\psi + \frac{1}{4}\omega_{ab}\Gamma^{ab}\psi,$$
$$\Xi = d\xi + \frac{1}{4}\omega_{ab}\Gamma^{ab}\xi + \frac{1}{4}\tilde{k}_{ab}\Gamma^{ab}\psi + \frac{1}{2l}e^{a}\Gamma_{a}\psi.$$

The non-vanishing components of an invariant tensor for the Maxwell superalgebra can be derived using the definitions of the S-expansion. Indeed, by Theorem VII.2 of Ref. [14], the invariant tensor of an S-expanded (super)algebra $\mathfrak{G}$ is given in terms of an invariant tensor of the original (super)algebra $\mathfrak{g}$ through

$$\left\langle T_{(A,\alpha)}T_{(B,\beta)}\right\rangle_{\mathfrak{G}} = \tilde{\alpha}_{\gamma}K_{\alpha\beta}{}^{\gamma}\left\langle T_{A}T_{B}\right\rangle_{\mathfrak{g}}, \tag{46}$$

where $\tilde{\alpha}_{\gamma}$ are arbitrary constants and $K_{\alpha\beta}{}^{\gamma}$ corresponds to a 2-selector. Thus, it is possible to show that the only non-zero components of a symmetric invariant tensor for the Maxwell superalgebra $s\mathcal{M}_3$ are given by

$$\left\langle J_{ab}J_{cd}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_0\left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_0\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{47}$$
$$\left\langle J_{ab}\tilde{Z}_{cd}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_2\left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_2\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{48}$$
$$\left\langle \tilde{Z}_{ab}\tilde{Z}_{cd}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_4\left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_4\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{49}$$
$$\left\langle J_{ab}Z_{cd}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_4\left\langle \tilde{J}_{ab}\tilde{J}_{cd}\right\rangle = \alpha_4\left(\eta_{ad}\eta_{bc}-\eta_{ac}\eta_{bd}\right), \tag{50}$$
$$\left\langle J_{ab}P_{c}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_2\left\langle \tilde{J}_{ab}\tilde{P}_{c}\right\rangle = \alpha_1\epsilon_{abc}, \tag{51}$$
$$\left\langle \tilde{Z}_{ab}P_{c}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_4\left\langle \tilde{J}_{ab}\tilde{P}_{c}\right\rangle = \left\langle J_{ab}\tilde{Z}_{c}\right\rangle = \alpha_3\epsilon_{abc}, \tag{52}$$
$$\left\langle P_{a}P_{b}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_4\left\langle \tilde{P}_{a}\tilde{P}_{b}\right\rangle = \alpha_4\eta_{ab}, \tag{53}$$
$$\left\langle Q_{\alpha}Q_{\beta}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_2\left\langle \tilde{Q}_{\alpha}\tilde{Q}_{\beta}\right\rangle = \left(\alpha_2-\alpha_1\right)C_{\alpha\beta}, \tag{54}$$
$$\left\langle Q_{\alpha}\Sigma_{\beta}\right\rangle_{s\mathcal{M}_3} = \tilde{\alpha}_4\left\langle \tilde{Q}_{\alpha}\tilde{Q}_{\beta}\right\rangle = \left(\alpha_4-\alpha_3\right)C_{\alpha\beta}, \tag{55}$$

where we have used eqs. (13)–(16) and the following definitions

$$\alpha_0 \equiv \tilde{\alpha}_0\mu_0, \quad \alpha_1 \equiv \tilde{\alpha}_2\mu_1, \quad \alpha_2 \equiv \tilde{\alpha}_2\mu_0$$
$$\alpha_3 \equiv \tilde{\alpha}_4\mu_1, \quad \alpha_4 \equiv \tilde{\alpha}_4\mu_0.$$

Considering (47)–(55) and the one-form connection (44) in the general expression for the Chern–Simons action (10) we can write the CS supergravity action for the minimal Maxwell superalgebra $s\mathcal{M}_3$ as

$$\begin{aligned} S_{CS}^{(2+1)} = k\int_{M}\Big[&\frac{\alpha_0}{2}\left(\omega^{a}{}_{b}d\omega^{b}{}_{a} + \frac{2}{3}\omega^{a}{}_{c}\omega^{c}{}_{b}\omega^{b}{}_{a}\right) \\ &+ \frac{\alpha_1}{l}\left(\epsilon_{abc}R^{ab}e^{c} - \bar{\psi}\Psi\right) + \alpha_2\left(R^{a}{}_{b}\tilde{k}^{b}{}_{a} + \frac{1}{l}\bar{\psi}\Psi\right) \\ &+ \frac{\alpha_3}{l}\left(\epsilon_{abc}\left(R^{ab}\tilde{h}^{c} + D_{\omega}\tilde{k}^{ab}e^{c}\right) - \bar{\xi}\Psi - \bar{\psi}\Xi\right) \\ &+ \alpha_4\left(R^{a}{}_{b}k^{b}{}_{a} + \frac{1}{l^2}e^{a}T_{a} + \frac{1}{l}\bar{\xi}\Psi + \frac{1}{l}\bar{\psi}\Xi\right) \\ &- d\left(\frac{\alpha_1}{2l}\epsilon_{abc}\omega^{ab}e^{c} + \frac{\alpha_3}{2l}\epsilon_{abc}\left(\tilde{k}^{ab}e^{c} + \omega^{ab}\tilde{h}^{c}\right)\right. \\ &\left.+ \frac{\alpha_2}{2}\omega^{a}{}_{b}\tilde{k}^{b}{}_{a} + \frac{\alpha_4}{2}\omega^{a}{}_{b}k^{b}{}_{a}\right)\Big], \end{aligned} \tag{56}$$

where $T^{a} = D_{\omega}e^{a}$ is the torsion 2-form. The Maxwell–Chern–Simons supergravity action (56) contains four sectors proportional to different arbitrary constants α_{γ}. The first term corresponds to the so-called exotic Lagrangian and it is Lorentz invariant [23,24]. The second term describes pure supergravity without cosmological constant. On the other hand, the terms proportional to α_2, α_3 and α_4 contain the coupling of the spin connection to the new gauge fields $\tilde{k}^{ab}$, k^{ab} and $\tilde{h}^{c}$. In particular the new Majorana spinor field ξ appears in the terms proportional to α_3 and α_4. One can see that the bosonic part of the action (56) contains the CS gravity action found in Refs. [17,25].

Let us note that the new fields appear also in the boundary term. Although the boundary terms have no contribution to the dynamics of the theory, they are an essential tool in the study of the AdS/CFT correspondence [26–29]. The inclusion of boundary contributions to (super)gravity has been extensively studied in Refs. [30–33].

Up to boundary terms, the full action (56) is invariant under the local gauge transformations of the generalized Maxwell supergroup,

$$\delta\omega^{ab} = D_{\omega}\rho^{ab}, \qquad \delta e^{a} = D_{\omega}\rho^{a} + e^{b}\rho_{b}{}^{a} + \bar{\epsilon}\gamma^{a}\psi, \tag{57}$$
$$\delta\tilde{k}^{ab} = D_{\omega}\tilde{\kappa}^{ab} - \left(\tilde{k}^{a}{}_{c}\rho^{b}{}_{c} - \tilde{k}^{bc}\rho^{a}{}_{c}\right) - \frac{1}{l}\bar{\epsilon}\gamma^{ab}\psi, \tag{58}$$
$$\begin{aligned}\delta k^{ab} = D_{\omega}\kappa^{ab} &- \left(k^{ac}\rho^{b}{}_{c} - k^{bc}\rho^{a}{}_{c}\right) - \left(\tilde{k}^{ac}\tilde{\kappa}^{b}{}_{c} - \tilde{k}^{bc}\tilde{\kappa}^{a}{}_{c}\right) \\ &+ \frac{2}{l^2}e^{a}\rho^{b} - \frac{1}{l}\bar{\varrho}\gamma^{ab}\psi - \frac{1}{l}\bar{\epsilon}\gamma^{ab}\xi,\end{aligned} \tag{59}$$
$$\delta\tilde{h}^{a} = D_{\omega}\tilde{\rho}^{a} + \tilde{h}^{b}\rho_{b}{}^{a} + \tilde{\kappa}^{a}{}_{c}e^{c} + \tilde{k}^{a}{}_{c}\rho^{c} + \bar{\varrho}\gamma^{a}\psi + \bar{\epsilon}\gamma^{a}\xi \tag{60}$$
$$\delta\psi = d\epsilon + \frac{1}{4}\omega^{ab}\gamma_{ab}\epsilon - \frac{1}{4}\rho^{ab}\gamma_{ab}\psi, \tag{61}$$
$$\begin{aligned}\delta\xi = d\varrho &+ \frac{1}{4}\omega^{ab}\gamma_{ab}\varrho + \frac{1}{2l}e^{a}\gamma_{a}\epsilon - \frac{1}{2l}\rho^{a}\gamma_{a}\psi - \frac{1}{4}\rho^{ab}\gamma_{ab}\xi \\ &+ \frac{1}{4}\tilde{k}^{ab}\gamma_{ab}\epsilon - \frac{1}{4}\tilde{\kappa}^{ab}\gamma_{ab}\psi,\end{aligned} \tag{62}$$

where the $s\mathcal{M}_3$ gauge parameter is given by

$$\begin{aligned}\rho = \frac{1}{2}\rho^{ab}J_{ab} &+ \frac{1}{2}\tilde{\kappa}^{ab}\tilde{Z}_{ab} + \frac{1}{2}\kappa^{ab}Z_{ab} + \frac{1}{l}\rho^{a}P_{a} + \frac{1}{l}\tilde{\rho}^{a}\tilde{Z}_{a} \\ &+ \frac{1}{\sqrt{l}}\epsilon^{\alpha}Q_{\alpha} + \frac{1}{\sqrt{l}}\varrho^{\alpha}\Sigma_{\alpha}.\end{aligned} \tag{63}$$

4. Comments and possible developments

In the present work we have derived the $D = 3$ $\mathcal{N} = 1$ Chern–Simons supergravity action from the minimal Maxwell superalgebra $s\mathcal{M}_3$. We have shown that the Maxwell supersymmetries can be obtained from the $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ superalgebra using the semigroup expansion procedure. The method considered here allowed to obtain the invariant tensor for the Maxwell superalgebra and to build the most general $D = 3$ CS supergravity action invariant under the Maxwell supergroup. The action describes a supergravity theory without cosmological constant in three dimensions and can be seen as a supersymmetric extension of the results in Refs. [17,25] where new extra fields have been added in order to have well defined S-expanded invariant tensors.

It is interesting to note that the minimal Maxwell superalgebra can also be derived as a generalized IW contraction of a minimal AdS–Lorentz superalgebra [34]. Analogously, the non-standard Maxwell superalgebra introduced in Refs. [35–37], can be recovered by performing a suitable IW contraction of the usual supersymmetric extension of the *AdS*–Lorentz algebra[1] [35,39]. Additionally, as shown in Refs. [40–42], the *AdS*–Lorentz (super)algebra can be alternatively obtained as an S-expansion of the *AdS* (super)algebra. Then, one could construct a CS supersymmetric action from the non-standard Maxwell superalgebra combining the S-expansion method with the IW contraction. Nevertheless, in this superalgebra the four-momentum generators P_a are not expressed as bilinears expressions of fermionic generators Q $\left(\left\{Q_\alpha, Q_\beta\right\} = -\frac{1}{2}\left(\Gamma^{ab}C\right)_{\alpha\beta} Z_{ab}\right)$. As a consequence, the supersymmetric action constructed out of the non-standard Maxwell superalgebra, shall not describe a supergravity action but an exotic alternative supersymmetric action.

Our results provide one more example of the usefulness of the Maxwell (super)symmetry in (super)gravity (see [5,6,13,17,25]). In particular, we have shown that the semigroup expansion procedure can be used in order to derive a new supergravity theory. The same procedure could be used in eleven dimensions in order to recover the CJS supergravity theory from a given superalgebra.

It would be interesting to analyze the boundary contributions present in the CS supergravity action in the *AdS*/*CFT* context. On the other hand, the results presented here could be useful in the construction of supergravity actions in higher dimensions. It seems that it should be possible to recover standard odd-dimensional supergravity from the Maxwell supersymmetries [work in progress].

A future work could be consider the $\mathcal{N}$-extended Maxwell superalgebras and their generalizations in order to build (p,q)-type CS supergravity models in a very similar way to the one introduced here. Eventually, the semigroup expansion could be useful in the construction of matter–supergravity theories.

Acknowledgements

This work was supported in part by FONDECYT Grant No. 1130653. Two of the authors (P.K.C., E.K.R.) were supported by grants from the Comisión Nacional de Investigación Científica y Tecnológica (CONICYT) and from the Universidad de Concepción, Chile. P.K.C. and E.K.R. wish to thank L. Andrianopoli, R. D'Auria and M. Trigiante for their kind hospitality at Dipartimieto Scienza Applicata e Tecnologia (DISAT) of Politecnico di Torino.

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[1] Also known as Poincaré semi-simple extended or $\mathfrak{so}(D-1,1) \oplus \mathfrak{so}(D-1,2)$ algebra. The semi-simple $o(N)$-extended superPoincaré algebra can be found in Ref. [38].